

22407

Quasilinear elliptic equations ...

S/042/61/016/001/001/007  
C 111/ C 533

$u(x_1, \dots, x_n)$  of the regular variational problem concerning the minimum of  $n$

$$I(u) = \int_{\Omega} F(x, u, u_{x_k}) dx_1 \dots dx_n$$

under the condition  $u|_S = \varphi(s)$ .

Let  $\Omega$  be a bounded domain of the  $x = (x_1, \dots, x_n)$  in the Euclidean  $E_n$ ;  $\Omega'$  -- strictly interior subdomain of  $\Omega$ ;  $C_{1,0}(\Omega)$  the set of all functions  $u(x)$  which are continuous with respect to  $x_k$  in the open  $\Omega$  together with the 1 first derivatives; let

$$|u|_{C_{1,0}(\Omega)} = \sum_{k=0}^1 \max_{x \in \Omega} |D^k u(x)|$$

be the norm. Let  $C_{1,\alpha}(\Omega)$  be the set of all functions from  $C_{1,0}(\Omega)$  for which

Card 2/13

22407

Quasilinear elliptic equations ...

S/042/61/016/001/001/007  
C 111/ C 333

$$\max_{\substack{x, x+h \in \Omega \\ |h| > 0}} \frac{|D^1 u(x+h) - D^1 u(x)|}{|h|^\alpha} = \Delta_{D^1 u}^\alpha$$

is bounded. The norm is:  $|u|_{C_{1,\alpha}(\Omega)} = |u|_{C_{1,0}(\Omega)} + \Delta_{D^1 u}^\alpha$ . Let  $C_0(\Omega)$  be the set of all functions continuous in  $\Omega$   $|u|_{C_0 \Omega} = \max_{x \in \Omega} |u(x)|$ . Let  $W_m^1(\Omega)$  and  $W_m^0(\Omega)$  be defined as usual (see V. J. Smirnov (Ref. 2: Kurs vysshey matematiki [Course in higher mathematics] t. IV, M., Fizmatgiz, 1959)).  $\max_{x \in \Omega} |u(x)|$  for  $u \in W_m^1(\Omega)$  is defined to be  $\max_{x \in \Omega} |u(x)|$ . Let  $D_1(\Omega)$  be the class of the functions  $u(x)$  which in  $\Omega$  possess  $l-1$  derivatives with respect to  $x_k$ , and for which the derivatives  $D^{l-1} u$  possess a differential in every point of  $\Omega$ . Let  $O_1(Q)$  be the class of the  $v(y_1, \dots, y_m) \in D_1(Q)$ , the  $l$ -th derivatives of which are bounded in every bounded domain of the  $y_1, \dots, y_m$ .

Card 3/13

22407

S/042/61/016/001/001/007

C 111/ C 333

Quasilinear elliptic equations ...

Let  $O(1)$  be the class of the functions measurable and bounded in every finite domain of the  $y_1, \dots, y_m$ . The statement "the norm  $|\cdot|$  is estimated by the data of the problem" means that the estimation is possible by the constants which occur in the conditions which are fulfilled by the problem.  $\mu_k(|u|)$  denotes positive nondecreasing and  $\nu_k(|u|)$  positive nonincreasing functions of  $|u|$  defined on  $[0, \infty)$  and finite for all finite  $|u|$ . The statement "the function  $f(x_1, \dots, x_n, u, p_1, \dots, p_n)$ ,  $x \in \Omega$  has the order of growth  $\leq m$  in  $p = \sqrt{\sum_{k=1}^n p_k^2}$ " says that  $\max_{x \in \Omega} |f(x, u, p_k)| \leq C(|u|)(p^2+1)^{m/2}$ . The boundary  $S$  possesses the property  $(A)$ , if there are  $a > 0$ ,  $0 < \theta < 1$  such that for every sphere  $K(\xi)$  with center on  $S$  and radius  $\xi \leq a$  it holds

$$\text{mes } [K(\xi) \cap \Omega] \leq (1 - \theta) \text{mes } K(\xi) .$$

Card 4/13

22407

Quasilinear elliptic equations ...

S/042/61/016/001/001/007  
C 111/ C 333

S belongs to  $C_{1,\alpha}$ ,  $\alpha \geq 0$ , if it can be covered by a finite number of open pieces, the equations of which belong to  $C_{1,\alpha}$ .

Theorem I. Let  $u(x)$  be a bounded generalized solution of

$$M_1(u) \equiv \frac{\partial}{\partial x_i} (a_i(x, u, u_{x_k})) + a(x, u, u_{x_k}) = 0 \quad (29)$$

i. e.  $u \in W_m^1(\Omega)$ ,  $|u| \leq M$  and  $u(x)$  is assumed to satisfy the inequality

$$\int_{\Omega} [a_i(x, u, u_{x_k}) \eta_{x_i} - a(x, u, u_{x_k}) \eta] dx = 0 \quad (30)$$

for arbitrary  $\eta(x) \in W_m^{01}(\Omega)$ . Let furthermore  $\max_{\Omega} |u_{x_i}| \leq M_1$ ,  $a_i(x, u, p_k) \in O_1(\Omega \times E_1 \times E_n)$  and  $a(x, u, p_k) \in O_0(\Omega \times E_1 \times E_n)$ . Let

$$\frac{\partial a_i(x+th, v, v_{x_k})}{\partial v_{x_j}} \xi_1 \xi_j \geq v_1 (|\nabla v|) v_2 (|\nabla v|) \sum_{i=1}^n \xi_i^2$$

Card 5/13

X

22407

Quasilinear elliptic equations ...

S/042/61/016/001/001/007  
C 111/ C 333

for  $v(x) = (1 - \tau) u(x) + \tau u(x + h)$ ,  $\tau \in [0, 1]$ ,  $x, x + h \in \Omega$ .  
The norm  $|u|_{C_{1,\alpha}(\Omega')}$ ,  $\alpha > 0$ , for arbitrary  $\Omega' \subset \Omega$  is then  
estimated by  $|u|_{C_{1,0}(\Omega')}$ . If, moreover,  $S \in C_{2,0}$  and  $\varphi(s) =$   
 $= u/S \in C_{2,0}(S)$ , then  $|u|_{C_{1,\alpha}(\Omega)}$  is estimated by  $|u|_{C_{1,0}(\Omega)}$  and  
 $|\varphi|_{C_{2,0}(S)}$ . If  $a_1$  and  $a$  belong as functions of their arguments to  
 $C_{1-1,\alpha}$  ( $1 \geq 2$ ) or to  $C_{1-2,\alpha}$  on every compact, while  $S$  and  $\varphi(s)$  belong  
to  $C_{1,\alpha}$ , then  $|u|_{C_{1,\alpha}(\Omega)}$  is estimated by  $|u|_{C_{1,0}(\Omega)}$  and by the  
data of the problem.  
The equation (29) is said to belong to the class  $(\exists)$ , if it satisfies  
for arbitrary  $\xi_1, \dots, \xi_n$  the conditions

Card 6/13

22407

S/042/61/016/001/001/007

C 111/ C 333

Quasilinear elliptic equations ...

$$\begin{aligned} \nu_1(|u|)(p^2 + 1)^{\frac{n-2}{2}} \sum_{i=1}^n \xi_i^2 &\leq a_{ij}(x, u, p_k) \xi_i \xi_j \leq \mu_1(|u|) \\ (p^2 + 1)^{\frac{n-2}{2}} \sum_{i=1}^n \xi_i^2 & \end{aligned} \quad (16)$$

$$|a(x, u, p_k)| \leq \mu_2(|u|) p^m + \mu_3(|u|) \quad (17)$$

and for large p

$$a_i(x, u, p_k) p_i \geq \nu_1(|u|) p^m \quad (m > 1), \quad (31)$$

$$\text{where } p^2 = \sum_{i=1}^n p_i^2.$$

Theorem II. For an arbitrary equation (29) of the class (3) the first boundary value problem with the boundary condition  $u|_S = \varphi(s)$  has at

Card 7/73

22407

S/042/61/016/001/001/007  
C 111/ C 333

Quasilinear elliptic equations ...  
least one solution in the class  $C_{2,\alpha}(\bar{\Omega}) \cap C_{3,\alpha}(\Omega)$ , if the maxima  
of the absolute values of the solutions  $u(x,\tau)$  of the boundary  
value problems

$$M_\tau(u) \equiv (1 - \tau) M_0(u) + \tau M_1(u) = 0, \quad u|_S = \tau \varphi, \quad \tau \in [0, 1]$$

are uniformly bounded, where  $K_\epsilon(u) \equiv \frac{\partial}{\partial x_i} F_{u_{x_i}}^0(u, u_{x_k}) - F_u^0(u, u_{x_k})$  and  
 $F^0(u, p_k) = (1 + p^2)^{m/2} + u^2$ . The coefficients  $a_i(x, u, p_k)$  and  $a(x, u, p_k)$   
must belong to  $C_{2,\alpha}$  and  $C_{1,\alpha}$  respectively as functions of their  
arguments on every compact. The boundary  $S$  and  $\varphi(s)$  must belong to  
 $C_{2,\alpha}$ .

Theorem III is a special case of theorem II.

Theorem IV. The propositions of theorem II are maintained, if all  
conditions except (31) are satisfied and if moreover the orders  
of growth in  $p$  of the functions  
Card 8/13

22407

Quasilinear elliptic equations ...

S/042/61/016/001/001/007  
C 111/ C 333

$\frac{\partial^2 a_i(x, u, p_k)}{\partial p_j \partial u}$ ,  $\frac{\partial^2 a_i(x, u, p_k)}{\partial u^2}$  and  $\frac{\partial a(x, u, p_k)}{\partial u}$  are not greater

than  $m-2-\varepsilon$ ,  $m-1-\varepsilon$  and  $m-\varepsilon$ , where  $\varepsilon > 0$  is arbitrary.

Theorem V. Let  $u(x) \in W_m^1(\Omega)$  be one of the generalized solutions of the variational problem

$$\inf I(u) = \inf \int_{\Omega} f(x, u, u_{x_k}) dx, \quad dx = dx_1 \dots dx_n, \quad (2)$$

$$u|_S = \varphi(s) \quad (3)$$

with the additional condition that all comparison functions are in the absolute value not greater than a constant  $M \geq \max_S |u|$ . This solution belongs to  $C_{0,\alpha}(\Omega)$ ,  $\alpha > 0$ , if

$$F(x, u, p_k) \in C_1(\Omega \times [-M, M] \times E_n)$$

Card 9/13



22407

Quasilinear elliptic equations ...

S/042/61/016/001/001/007  
C 111/ C 333

X

$$F_{p_1}(x, u, p_k) p_1 \geq \nu_1(|u|) p^m \quad \text{for } p \gg 1$$

and

$$p \sum_{i=1}^n |F_{p_i}(x, u, p_k)| + |F_u(x, u, p_k)| \leq \mu_1(|u|) (p^m + 1).$$

Under the same assumptions on  $F$ , every bounded  $u(x) \in W_m^1(\Omega)$ , which gives  $I$  a stationary value belongs to  $C_{0,\infty}(\Omega)$ . If, moreover, the boundary of  $\Omega$  satisfies the condition (A), and if  $\varphi(s)$  can be continued in  $\Omega$  so that  $\varphi(x) \in O_1(\Omega)$ , then in both cases it holds  $u(x) \in C_{0,\infty}(\bar{\Omega})$ .

Theorem VI. If only the natural restrictions 1.) - 4.) are satisfied for  $F(x, u, p_k)$ , then every bounded generalized solution  $u(x) \in W_m^1(\Omega)$

Card 10/13

22407

S/042/61/016/001/001/007

C 111/ C 333

Quasilinear elliptic equations ...

of the variational problem (2), (3) belongs to  $C_{k,\alpha}(\bar{\Omega})$ ,  $\alpha > 0$ , if  $F(x,u,p_k)$  as function of its arguments belongs to  $C_{k,\alpha}$ ,  $k \geq 3$  on every compact. If, moreover,  $S \in C_{1,\alpha}$  and  $\varphi \in C_{1,\alpha}$ ,  $2 \leq 1 \leq k$ , then  $u(x)$  belongs to  $C_{1,\alpha}(\bar{\Omega})$  too. As natural restrictions for  $F(x,u,p_k)$  there are denoted:

- 1.)  $\nu_1(|u|)(p^2 + 1)^{m/2} \leq F(x,u,p_k) \leq \mu_1(|u|)(p^2 + 1)^{m/2}$
- 2.) The Euler equation for  $F(x,u,p_k)$  is uniformly elliptic. ((1) is called uniformly elliptic, if (16) holds).
- 3.)  $F$  is sufficiently smooth, where the differentiation of  $F$  and of its partial derivatives with respect to  $p_k$  reduces the order of growth of  $F$  and of the derivatives mentioned at least by 1, while the differentiation with respect to  $x_k$  and  $u$  does not increase these orders of growth.

Card 11/13

22407

Quasilinear elliptic equations ...

S/042/61/016/001/001/007  
C 111/ C 333

For all sufficiently large  $p$  it holds

$$F_{p_1}(x, u, p_k) p_1 \geq v_2(|u|) p^m.$$

The given theorems are the main results of the paper; 25 theorems and 11 lemmata are proved.

The author mention: V. J. Kazimirov, A. G. Sigalov, A. J. Koshelev, G. J. Shilova, S. L. Sobolev, V. J. Plotnikov, A. D. Aleksandrov, A. V. Pogorelov, Ye. P. Sen'kin, J. Ya. Bakel'man.

There are 16 Soviet-bloc and 25 non-Soviet-bloc references. The four most recent references to English-language publications read as follows: L. Nirenberg, Estimates and existence of solutions of elliptic equations, Commun. Pure and Appl. Math. 2, 3(1956), 509-531;

Card 12/13

Quasilinear elliptic equations

22407  
S/042/61/016/001/001/007  
C 111/,C 333

J. Nash, Continuity of solutions of parabolic and elliptic equations, Amer. Journ. Math. 80, No. 4 (1958), 931-954; R. Finn and D. Gilbarg, Three-dimensional subsonic flows, and asymptotic estimates for elliptic partial differential equations, Acta math. 98 (1957), 265-296; C. B. Morrey, Second order elliptic equations in several variables and Hölder Continuity, Math. Z. 72 (1959), 146-164.

SUBMITTED: July 12, 1960

Card 13/13

23799

16,3500

S/020/61/138/001/003/023  
C 111/ C 222

AUTHORS: Ladyzhenskaya, O. A. and Ural'tseva, N. N.

TITLE: Differential properties of bounded generalized solutions to n-dimensional quasilinear elliptic equations and variation problems

PERIODICAL: Akademiya nauk SSSR. Doklady, v. 138, no. 1, 1961, 29-32

TEXT: The authors investigate the equation

$$\sum_{i=1}^n \frac{\partial}{\partial x_i} (a_i(x, u, u_x)) + a(x, u, u_x) = 0 \quad (1)$$

where  $a_i$  and  $a$  are measurable functions satisfying

$$\begin{aligned} \|a_i(x, u, p_j)\|_{p_j} + \|a(x, u, p_j)\|_{p_j} &\leq M(|u|)(1 + p)^{\alpha}, \\ a_i(x, u, p_j) p_j &\geq \psi(|u|) p^{\beta} - \varphi(|u|), \end{aligned} \quad (2)$$

Card 1/6

23799

S/020/61/138/001/003/023

C 111/ C 222

Differential properties of ...

where  $m > 1$  and  $p = \sum_{j=1}^n p_j^2$ . Let besides the condition

$$\begin{aligned} & \psi(u)(1+p)^{m-2} \sum_{i=1}^n \left( \frac{2}{p_i} \leq \frac{\partial a_1(x, u, p_j)}{\partial p_i} \right) \leq \psi(u)(1+p)^{m-2} \sum_{i=1}^n \left( \frac{2}{p_i} \right) \\ & \left| \frac{\partial a_1}{\partial p_j} \right| p^2 + \left| \frac{\partial a_1}{\partial u} \right| p + \left| \frac{\partial a}{\partial p_1} \right| p + \left| \frac{\partial a}{\partial u} \right| \leq \psi(u)(1+p)^m \end{aligned} \quad (3)$$

be satisfied incidentally, where  $\psi(t)$  is monotone non-increasing,  $\psi(t)$  -- monotone non-decreasing,  $\psi(t)$  and  $\psi'(t) > 0$ ,  $t \geq 0$ .

A function  $u(x) \in W_m^1(\Omega)$  for which

$$I(u, \gamma) = \int_{\Omega} [a_1(x, u, u_x) \gamma_{x_1} - a(x, u, u_x) \gamma] dx = 0 \quad (4)$$

holds for every bounded function  $\gamma$  of  $W_m^1(\Omega)$  is called a generalized

Card 2/6

23799

Differential properties of ...

S/020/61/138/001/003/023

C 111/ C 222

solution of (1).

Lemma 1: For the bounded generalized solution  $u(x)$  of (1) there hold the inequalities

$$\int_{K(\rho)} |\nabla u|^m dx \leq c \rho^{n-m+\alpha} \quad (5)$$

$$\int_{K(\rho)} |x - y|^{-n+m-\alpha/2} |\nabla u|^m dx \leq c \rho^{\alpha/2} \quad (6)$$

where  $K(\rho)$  is an arbitrary sphere of radius  $\rho$  in  $\Omega$ , and the constant  $c$  depends only on  $\mu(\max |u|)$ ,  $\nu(\max |u|)$  of (2).

Lemma 2: Every bounded generalized solution  $u(x)$  of (1) with  $m \geq 2$  satisfies

$$\int_{K(\rho)} (1 + |\nabla u|)^m dx \leq c \rho^{\alpha} \int_{K(\rho)} (1 + |\nabla u|)^{m-2} |\Delta u|^2 dx \quad (7)$$

for every bounded  $\mathbb{F}$  of  $\mathbb{W}_m^1(K(\rho))$ , where the constant  $c$  depends only on  $\mu(\max |u|)$  and  $\nu(\max |u|)$  of (2).

Card 3/6

23799

Differential properties of ...

S/020/61/138/001/003/023  
C 111/ C 222

Lemma 2': If  $b(x) > 0$ , and if for every  $\varrho > 0$  and  $y \in \Omega$  it holds

$\int_{\Omega} |x-y|^{-n+m-1/2} b^m(x) dx \leq c_1 \varrho^{m/2}$ ,  $c_1 > 0$ ,  $1 \leq m \leq 2$  then it holds

$$\int_{\Omega} b^m |f|^2 dx \leq c_2 \varrho^{2m/m} \int_{\Omega} b^{m-2} |f|^2 dx \quad (8)$$

where  $f$  is an arbitrary bounded function of  $\dot{W}_B^1(K(\varrho))$ , and the constant  $c$  depends only on  $c_1, n, m$ .

From lemma 2' it follows that lemma 2 holds also for  $1 \leq m \leq 2$ .

Theorem 1: The uniqueness theorem in the small holds for a bounded generalized solution  $u(x)$  of (1) i. e.: two bounded generalized solutions  $u'(x)$  and  $u''(x)$  being equal on the surface of  $K(s)$  are identical in  $K(s)$  if only the radius  $\varrho$  is smaller than a certain number which is determined by  $\max_{\Omega} (|u'|, |u''|)$  and  $\max_{\Omega} (|u'|, |u''|)$  of (2) and (3).

Theorem 2: If (2) and (3) are satisfied then every bounded generalized

Card 4/6



23799

Differential properties of ...

S/020/61/138/001/003/023  
C 111/ C 222

solution  $u(x)$  of (1) has generalized second derivatives and satisfies (1) almost everywhere. For this solution it holds

$$\int_{\Omega'} [|\nabla u|^{m+2} + (1 + |\nabla u|)^{m-2} \sum_{i,j} u^2_{x_i x_j}] dx < c \in \Omega' \quad (10)$$

where  $\Omega'$  is an arbitrary strongly inner subregion of  $\Omega$ . If  $S$  and  $\varphi = u/s$  are two times continuously differentiable then (10) holds for  $\Omega' = \Omega$  too.

Let

$$J(u) = \int_{\Omega} F(x, u, u_x) dx, \quad u|_S = \varphi \quad (12)$$

Theorem 3: Every bounded  $u(x)$  of  $W^1_m(\Omega)$  for which

$$\delta J(u) = \int_{\Omega} (F_{u_{x_i}}(x, u, u_x) \eta_{x_i} + F_u \eta) dx = 0 \text{ holds for every bounded}$$

$\eta(x) \in W^{0,1}_m(\Omega)$ , belongs  $C_{k,\alpha}(\Omega)$  ( $k \geq 3, \alpha > 0$ ) if  $F(x, u, p_j)$  as a function

Card 5/6

23799

S/020/61/138/001/003/023  
C 111/ C 222

Differential properties of ...  
of all arguments belongs to  $K_{\text{loc}}$  and satisfies only the "natural"  
assumptions of (Ref. 1: O. A. Ladyzhenskaya, N. N. Ural'tseva, DAN  
135, no. 6 (1960); Ref. 2: O. A. Ladyzhenskaya, N. N. Ural'tseva, Usp.  
matem. nauk, 16, no. 1 (1961)).

There are 4 Soviet-bloc and 2 non-Soviet-bloc references.

ASSOCIATION: Leningradskiy gosudarstvennyy universitet imeni A. A.  
Zhdanova (Leningrad State University imeni A. A.  
Zhukov)

PRESENTED: December 24, 1960, by V. J. Smirnov, Academician

SUBMITTED: December 20, 1960

Card 6/6

LADYZHENSKAYA, O.A.; URAL'TSEVA, N.N.

Boundary value problem for linear and quasi-linear parabolic  
equations. Dokl. AN SSSR 139 no.3: 544-547 J1 '61 (MIRA 14:7)

1. Leningradskiy gosudarstvennyy universitet im. A.A. Zhdanova.  
Predstavleno akademikom V.I. Smirnovym.  
(Boundary value problems)  
(Differential equations, Linear)

LADYZHENSKAYA, O.A.; URAL'TSEVA, N.N.

Regularity of generalized solutions of quasi-linear elliptic equations. Dokl. AN SSSR 140 no.1:45-47 S.O '61. (MIRA 14:9)

1. Leningradskoye otdeleniye Matematicheskogo instituta im. V.A. Steklova AN SSSR. Predstavleno akademikom V.I. Smirnovym.  
(Differential equations)

LADYZHENSKAYA, O. A.

"Quasilinear equations of elliptic and parabolic types"

report submitted at the Intl Conf on Mathematics, Stockholm, Sweden,  
15-22 Aug 62

LADYZHENSKAYA, O. A.

"Sur les equations differentiel les quasi lineaires de type elliptique et parabolique."

Report to be submitted for the International Colloquim on Partial Differential Equations (CNRS) Paris France, 25-30 June 1962.

33628

8/038/62/026/001/001/003  
B112/B108

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AUTHORS: Ladyzhenskaya, O. A., and Ural'tseva, N. N.

TITLE: Boundary value problem for linear and quasi-linear parabolic equations. I.

PERIODICAL: Akadimiya nauk SSSR. Izvestiya seriya Matematicheskaya, v. 26, no. 1, 1962, 5-52

TEXT: For linear parabolic equations of the form  

$$Lu = u_t - (\partial/\partial x_i)(a_{ij}(x,t)u_{x_j} + a_i(x,t)u + f_i(x,t)) + b_i(x,t)u_{x_i}$$

+  $a(x,t)u + f(x,t) = 0$   
 with unbounded coefficients, estimates of the Hölder norm of the solutions and of their derivatives are derived. For the solutions of general quasi-linear parabolic equations  

$$\mathcal{L}u = u_t - (\partial/\partial x_i)(a_i(x,t,u,u_{x_k})) + a(x,t,u,u_{x_k}) = 0$$

"with a divergent right-hand side", apriori estimates are obtained. By means of these estimates it is demonstrated that the first boundary value

Card 1/2

33628

S/038/62/026/001/001/003  
B112/B108

Boundary value problem for...

problem for such equations can be solved "in the large". All results are new inclusive that for the case of a single spatial variable. The conditions under which the apriori estimates are obtained and under which the solvability "in the large" is demonstrated are not only sufficient but in a certain sense also necessary. There are 37 references: 21 Soviet-bloc and 16 non-Soviet-bloc. The four references to English-language publications read as follows: Nash J., Continuity of solutions of parabolic and elliptic equations, Amer. J. Math., 80 (1958), 931-954; Friedman A., On quasi-linear parabolic equations of the second order, J. Math. and Mech., 7, No. 5 (1958), 771-791; 793-809, Morrey C. B., Second order elliptic equations in several variables and Hölder continuity, Math. Z., 72 (1959), 146-164; Friedman A., Boundary estimates for second order parabolic equations and their applications, Journ. Math. and Mech., 7, No. 5 (1958), 771-791. X

SUBMITTED: May 18, 1961

Card 2/2



S/038/62/026/005/003/003  
B112/B186

AUTHORS: Ladyzhenskaya, O. A., and Ural'tseva, N. N.  
TITLE: Boundary value problems for linear and quasi-linear parabolic equations. II  
PERIODICAL: Akademiya nauk SSSR. Izvestiya. Seriya matematicheskaya, v. 26, no. 5, 1962, 753-780

TEXT: The first boundary value problem for quasi-linear parabolic equations

$$\mathcal{L}u = u_t - \sum_{i=1}^n a_i(x, t, u, u_{x_k}) / dx_i + a(x, t, u, u_{x_k}) = 0 \quad (1)$$

with "divergent main part" is considered from a global point of view. Local results concerning such equations have been obtained in the first part of this paper (Izvestiya Ak. nauk SSSR, seriya matemat., 26 (1962), 5-52). Global estimates of  $|Vu|$  and of the Hölder norm of  $u_{x_k}$  are derived. From these estimates, the existence of classical solutions is

Card 1/2

Boundary value problems for...

S/038/62/026/005/003/003  
B112/B186

proved for bounded and unbounded domains and, in particular, for  
Cauchy's problem. Special attention is paid to the theorem of existence  
at an arbitrary growth, with respect to problems of subsurface hydro-  
dynamics.

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Card 2/2

S/020/62/147/001/002/022  
B112/B102

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AUTHORS:

TITLE:

Ladyzhenskaya, O. A., Ural'tseva, N. N.

The first boundary value problem for quasilinear second-order parabolic equations of the general form

PERIODICAL: Akademiya nauk SSSR. Doklady, v. 147, no. 1, 1962, 28-30

TEXT: The parabolic boundary value problem

$$u_t - \sum_{i,j=1}^n a_{ij}(x,t,u,u_{x_k})u_{x_i x_j} + a(x,t,u,u_{x_k}) = 0, \quad (1)$$

is considered under the following genuine "restrictions":

$$(a) \quad \begin{cases} u|_S = 0, & u|_{t=0} = \varphi(x). \\ a(x,t,u,0) \geq -b_1 u^2 - b_2, & b_i = \text{const} \geq 0, \\ \sum_{i,j=1}^n a_{ij}(x,t,u,0) \delta_{ij} \geq 0 \end{cases}$$

for  $(x,t) \in \bar{Q}_T = \bar{\Omega} \times [0 \leq t \leq T]$  and any  $u$ ;

Institut im. A. A. Zhdanova  
imeni A. A. Zhdanov)

PRESE

SUBMIT

Card 2/

Card 1/2

.. I. Smirnov, Academician

42536  
S/020/62/147/001/002/022  
B112/B102

16.3500

AUTHORS: Ladyzhenskaya, O. A., Ural'tseva, N. N.

TITLE: The first boundary value problem for quasilinear second-order parabolic equations of the general form

PERIODICAL: Akademiya nauk SSSR.. Doklady, v. 147, no. 1, 1962, 28-30

TEXT: The parabolic boundary value problem

$$u_t - \sum_{i,j=1}^n a_{ij}(x,t,u,u_{x_k}) u_{x_i x_j} + a(x,t,u,u_{x_k}) = 0, \quad (1)$$

$$u|_S = 0, \quad u|_{t=0} = \varphi(x). \quad (2)$$

is considered under the following genuine "restrictions":

(a)  $a(x,t,u,0) \geq -b_1 u^2 - b_2, \quad b_i = \text{const} \geq 0,$

$$\sum_{i,j=1}^n a_{ij}(x,t,u,0) \xi_i \xi_j \geq 0$$

for  $(x,t) \in \bar{Q}_T = \bar{\Omega} \times [0 \leq t \leq T]$  and any  $u$ ;

Card 1/2

S/020/62/147/001/002/022  
B112/B102

The first boundary value problem...

$$(b) \quad v(|u|)(1+p)^{m-2} \sum_{i=1}^n \xi_i^2 \leq a_{ij}(x, t, u, p_k) \xi_i \xi_j \leq \mu(|u|)(1+p)^{m-2} \sum_{i=1}^n \xi_i^2,$$

$$|a| + \left| \frac{\partial a}{\partial u} \right| + \left| \frac{\partial a}{\partial p_k} \right| (1+p) + \left| \frac{\partial a}{\partial x_k} \right| + \left| \frac{\partial a_{ij}}{\partial u} \right| (1+p)^2 +$$

$$+ \left| \frac{\partial a_{ij}}{\partial p_k} \right| (1+p)^3 + \left| \frac{\partial a_{ij}}{\partial x_k} \right| (1+p)^3 \leq \mu(|u|)(1+p)^m,$$

where  $m$  is an arbitrary number, for  $(x, t) \in \bar{Q}_T$  and any  $u, p_k$ . For the general equation (1), the same solution estimates are derived as in previous papers for a parabolic equation "with divergent principal part" (cf. O. A. Ladyzhenskaya, DAN, 107, No. 5 (1956); Tr. Mosk. matem. obshch., 7, 149 (1958), and O. A. Ladyzhenskaya, N. N. Ural'tseva, UMN, 16, no. 1, 19 (1961)). The derivation departs from the previous ones.

ASSOCIATION: Leningradskiy gosudarstvennyy universitet im. A. A. Zhdanova  
(Leningrad State University imeni A. A. Zhdanov)

PRESENTED: May 21, 1962, by V. I. Smirnov, Academician

SUBMITTED: May 17, 1962

Card 2/2

LADYZHENSKAYA, O. A.; URAL'TSEVA, N. N.

On possible extensions of the concept of solution for linear  
and quasi-linear second-order elliptic equations. Vest. LGU 18  
no.1:10-25 '63. (MIRA 16:1)

(Differential equations)

45652

S/038/63/027/001/004/004  
B112/B186

163580

AUTHORS:

Ladyzhenskaya, O. A., and Ural'tseva, N. N.

TITLE:

Boundary-value problem for linear and quasilinear equations and systems of the parabolic type. III

PERIODICAL:

Akademiya nauk SSSR. Izvestiya. Seriya matematicheskaya, v. 27, no. 1, 1963, 161-240

TEXT: General quasilinear equations

$$\mathcal{L}u \equiv u_t - \sum_{i,j=1}^n a_{ij}(x, t, u, u_{x_i}) u_{x_i x_j} + a(x, t, u, u_{x_i}) = 0, \quad (1)$$

and parabolic systems

$$u_t^l - \sum_{i,j=1}^n \frac{d}{dx_i} \left( \sum_{j=1}^n a_{ij}(x, t) u_{x_j}^l + \sum_{m=1}^N d_{im}^l(x, t) u^m + f_i(x, t) \right) +$$

$$+ \sum_{i=1}^n \sum_{m=1}^N b_{im}^l(x, t) u_{x_i}^m + \sum_{m=1}^N b^{lm}(x, t) u^m + f^l(x, t) = 0, \quad l=1, \dots, N, \quad (2)$$

$$u_t^l - \sum_{i,j=1}^n a_{ij}(x, t, u^m) u_{x_i x_j}^l + a^l(x, t, u^m, u_{x_i}^m) = 0, \quad l=1, \dots, N, \quad (3)$$

Card 1/2

Card

LADYZHENSKAYA, O.A.; URAL'TSEVA, N.N.

Holder continuity of solutions and their derivatives for linear  
and quasi-linear elliptic and parabolic equations. Dokl. AN SSSR  
155 no.6:1258-1261 Ap '64. (MIRA 17:4)

1. Leningradskoye otdeleniye Matematicheskogo instituta im. V.A.  
Steklova AN SSSR. Predstavleno akademikom V.I.Smirnovym.



ACCESSION NR: AP4034025

S/0020/64/155/006/1258/1261

AUTHOR: Ladyzhenskaya, O. A.; Ural'tseva, N. N.

TITLE: On Hölder-continuity of solutions, and derivatives of solutions, of linear and quasi-linear equations of elliptic and parabolic type.

SOURCE: AN SSSR. Doklady\*, v. 155, no. 6, 1964, 1258-1261

TOPIC TAGS: partial differential equation, second order, elliptic equation, elliptic system, parabolic equation, parabolic system, generalized solution

ABSTRACT: In a series of (seven) earlier papers the authors have studied equations of elliptic or parabolic type, of the forms

$$\mathcal{L}_1 u \equiv \frac{\partial}{\partial x_i} (a_{ij}(x) u_{x_j} + a_i(x) u) + b_i(x) u_{x_i} + c(x) u = f(x), \quad (1)$$

$$\mathcal{L}_2 u \equiv \frac{\partial u}{\partial t} - \frac{\partial}{\partial x_i} (a_{ij}(x, t) u_{x_j} + a_i(x, t) u) + b_i(x, t) u_{x_i} + c(x, t) u = f(x, t), \quad (2)$$

$$\mathcal{L}_3 u \equiv \frac{\partial}{\partial x_i} (a_i(x, u, u_x)) + a(x, u, u_x) = 0, \quad (3) \quad \mathcal{L}_4 u \equiv u_t - a_{ij}(x, t, u, u_x) u_{x_i x_j} + a(x, t, u, u_x) = 0 \quad (4)$$

$$\mathcal{L}_5 u \equiv \frac{\partial u}{\partial t} - \frac{\partial}{\partial x_i} (a_i(x, t, u, u_x)) + a(x, t, u, u_x) = 0, \quad (5) \quad \mathcal{L}_6 u \equiv a_{ij}(x, u, u_x) u_{x_i x_j} + a(x, u, u_x) = 0, \quad (5)$$

Card 1/4

ACCESSION NR: AP4034025

and certain systems of such equations. One of the main objects of their work was investigating the Hölder-continuity of the solutions and their derivatives, as well as getting estimates for their Hölder norms in terms of constants depending on the coefficient functions. By constructing special examples, they have shown that in a certain sense, their results cannot be improved. Assuming that the solutions under consideration are bounded and have a certain degree of smoothness, it was shown that every solution  $u$  of equations (1) - (4) as well as each  $u_{x_i}$  belong to a certain class  $B$ ; the gradient with respect to  $x$  of every solution of (5) or (6) belongs to a certain class  $B^N$ . (A function belongs to such a class if it satisfies certain inequalities involving free parameters.) Then it was proved that the functions in the various  $B$  classes are Hölder-continuous and that their Hölder norm can be estimated in terms of the numerical parameters defining  $B$ . The object of this paper is to present a shorter method of proof, by-passing the study of the  $B$ -classes. The reasoning is based on lemmas from the earlier papers and a new lemma, concerning functions in the class  $W_2^1(K_2)$ , where  $K_2 = \{(x) \leq 2\}$ . Since the results are those which were presented earlier, they are not re-stated here. Instead, the method is illustrated on the example

$$u_t - \frac{\partial}{\partial x_i} (a_{ij}(x, t) u_{x_j}) = 0 \quad (7)$$

Card 2/4

ACCESSION NR: AP4034025

to which corresponds the integral identity

$$\int (u\eta + a_{ij}u_{x_j}\eta_{x_i}) dx = 0, \quad (8)$$

where  $\eta$  is a smooth function, finite in the region under consideration. The main part of the argument consists in showing that if a solution  $u(x,t)$  of (7) is defined in the cylinder  $Q_2 = K_2 \times [0, a]$  and if its range is  $[0, 1]$ , then

$$\text{osc}(u, Q_1) < \eta \text{osc}(u, Q_2) = \eta, \quad (10)$$

where  $Q_1$  is the cylinder  $K_1 \times [3/4a, a]$ ,  $K_1 = \{|x| \leq 1\}$ . Then the full statement [too long to be repeated here] of the result for (generalized) solutions of (3) is given, followed by an outline of the method to be used in the case of equations (5) and (6). Orig. art. has: 16 equations.

ASSOCIATION: Leningradskoye otdelenie Matematicheskogo instituta im. V. A. Steklova Akademii nauk SSSR (Leningrad Division of the Mathematics Institute Academy of Sciences, SSSR)

SUBMITTED: 18Dec63

ENCL: 00

Card 3/4

ACCESSION NR: AP4034025

SUB CODE: MA

NO REF SOV: 001

OTHER: 001

Card 4/4

L 11460-65 EWT(d) Pg-4 IJP(e)/ASD(a)-5/AFWL/SSD/ESD(dp)/ESD(gs)/ESD(t)

ACCESSION NR: AP4046364

S/0020/64/158/003/0513/0515 B

AUTHORS: Ladyzhenskaya, O. A.; Rivkind, V. Ya.; Ural'tseva, N. N.

TITLE: Classical solvability of diffraction problems for equations of the elliptical and parabolic type

SOURCE: AN SSSR. Doklady\*, v. 158, no. 3, 1964, 513-515

TOPIC TAGS: diffraction analysis, boundary value problem, elliptic differential equation, parabolic differential equation, existence theorem

ABSTRACT: In an earlier paper, one of the authors (Ladyzhenskaya, DAN, 96, No. 3, 433, 1954) proved that diffraction problems can be reduced to standard boundary and initial-boundary problems, for which various solution methods are available, thereby proving the solvability of diffraction problems. Furthermore, it was pointed out that more accurate to the diffraction problems can be obtained by

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ACCESSION NR: AP4046364

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making more precise the formulation of the corresponding boundary and initial-boundary value problems. It was pointed out, however, that the results obtained for elliptic and parabolic equations are quite crude. Following a later development of new methods for the investigation of differential properties of generalized solutions (Ladyzhenskaya and Ural'tseva, Izv. AN SSSR ser. matem. v. 26, No. 1, 5, 1962; UMN, v. 26, No. 1, 19, 1961) which led to more accurate relationships between the differential properties of the generalized solutions of elliptic and parabolic equations and the differential properties of the coefficients of the equation, it has become possible to refine the results for elliptic and parabolic diffraction problems. Two problems of this type are solved by way of an example and several theorems proved concerning the solvability of these problems. This report was presented by V. I. Smirnov. Orig. art. has: 14 formulas.

ASSOCIATION: Leningradskoye otdeleniye Matematicheskogo instituta

Card 2/3

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ACCESSION NR: AP4046364

Im. V. A. Steklova Akademii nauk SSSR (Leningrad Division, Mathematics Institute, Academy of Sciences SSSR)

SUBMITTED: 15Apr64

ENCL: 00

SUB CODE: MA

NR REF SOV: 009

OTHER: 000

Cord 3/3

LADYZHENSKAYA, Ol'ga Aleksandrovna; URAL'TSEVA, Nina Nikolayevna;  
SOLOMYAK, M.Z., red.

[Linear and quasilinear elliptic equations] Lineinye i kvazilineinye uravneniia ellipticheskogo tipa. Moskva, Nauka, 1964. 538 p. (MIRA 18:1)



LADYZHENSKAYA, O.A.

In memory of Vladimir Andreevich Steklov, 1864-1926 on the  
100th anniversary of his birth. Trudy Mat. inst. 73:3-4 '64.  
(MIRA 18:3)

L 63358-65 EWT(d) IJP(c)

ACCESSION NR: AT5018142

UR/2517/64/073/000/0172/0220

AUTHOR: Ladyzhenskaya, O. A.; Ural'tseva, N. N.

TITLE: On Hölder continuity of solutions and their derivatives for linear and quasi-linear elliptic and parabolic equations/6

SOURCE: AN SSSR, Matematicheskii institut. Trudy, v. 73, 1964. Krayevyye zadachi matematicheskoy fiziki (Boundary value problems in mathematical physics); sbornik rabot, no. 2, 172-220

TOPIC TAGS: partial differential equation, boundary value problem, elliptic differential equation, parabolic equation

ABSTRACT: Hölder continuity criteria are studied for the solutions of elliptic and parabolic linear equations, quasi-linear equations having a divergent principal part, and general quasi-linear equations. Estimates are found for Hölder constants for the solutions with their derivatives in terms of the constants of coefficient functions of the equations and the maxima of the solutions and their derivatives. For finding the Hölder constants, the approach is as follows. It is first proved that a solution or gradient of a solution belongs to a certain class. The proof is based on the choice of arbitrary functions satisfying certain integral equalities and

Card 1/2

L 63358-65

ACCESSION NR: AT5018142

on the application of Hölder and Jung inequalities. It is then proved that these  $\mathcal{G}$ -class functions are Hölder continuous and that their Hölder constants may be estimated in terms of numerical parameters used to define the classes  $\mathcal{G}$ . The difficulties encountered in this proof make it desirable to develop a simpler method for determining the Hölder constants of the solutions of these equations. On the basis of previous work by the authors and of a method due to Moser involving the use of not only the solution itself, but also the logarithm of the solution, the so-called "sub-solution," this simpler method is offered. Orig. art. has: 257 formulas.

ASSOCIATION: none

SUBMITTED: 00

ENCL: 00

SUB CODE: HA

NO REF SOV: 004

OTHER: 003

Card 2/2

SOLONNIKOV, V.A.; PETROVSKIY, I.G., akademik, otv. red.; NIKOL'SKIY,  
S.M., prof., zamestitel' otv. red.; LADYZHENSKAYA O.A., red.

[Boundary value problems for linear parabolic systems of  
differential equations of the general type.] O Krasvykh  
zadachakh dlia lineinykh parabolicheskikh sistem differentsial'-  
nykh uravnenii obshchego vida. Moskva, Nauka, 1965. 162 p.  
(Akademiia nauk SSSR. Matematicheskii institut. Trudy, vol.83)  
(MIRA 18:11)

L 7914-66 EMT(d) IJP(c)  
 ACC NR: AP5027355 <sup>44,55</sup> SOURCE CODE: UR/0043/65/000/004/0038/0046  
 AUTHORS: Ladyzhenskaya, O. A.; Stupyalis, L. <sup>44,55</sup>  
 ORG: none 44  
B  
 TITLE: Equations of mixed type  
 SOURCE: Leningrad. Universitet. Vestnik. Seriya matematiki, mekhaniki i astronomii, no. 4, 1965, 38-46 <sup>44,55</sup>  
 TOPIC TAGS: <sup>14,44,55</sup> differential equation, <sup>16,44,55</sup> partial differential equation, <sup>14,44,55</sup> elliptic equation, <sup>16,44,55</sup> hyperbolic equation, <sup>16,44,55</sup> parabolic equation  
 ABSTRACT: The authors consider the problem of determining  $u(x,t)$ , satisfying one of  $L_i^{(j)} u = f_i^{(j)}$ ,  $i = 1,2,3$  for each  $j = 1,2$  on  $\Omega_j \times [0,T]$ , and various initial conditions depending on  $i$ . Here  $i$  indicates an elliptic, parabolic, or hyperbolic type of equation. Conjugacy conditions on the common boundary of  $\Omega_1$  and  $\Omega_2$  are to be satisfied. The method of solution is illustrated on  
 Card 1/2 UDC: 517.946  
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L 7914-66  
AOC NR: 7 AP5027355

$$\begin{array}{l} -\Delta u = f_1(x, t), \\ u_t - \Delta u = f_2(x, t), \\ u_{tt} - \Delta u = f_3(x, t), \end{array} \quad \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

under simple conjugacy conditions. Orig. art. has: 39 formulas.

SUB CODE: MA/ SUBM DATE: 24Apr64/ ORIG REF: 007

Card 2/2 (911)

LADYZHENSKAYA, O.A.; STUPYALIS, L.

Mixed type equations. Vest. IGU 20 no.19:38-46 '65.

(MIRA 18:10)

L 20741-66 EWT(d) IJP(c)

ACC NR: AP6010422

SOURCE CODE: UR/0020/66/167/002/0309/0311

AUTHOR: Krzhevitski, A.; Ladyzhenskaya, O. A.

ORG: none

TITLE: A method of nets for the Navier-Stokes equations

SOURCE: AN SSSR. Doklady, v. 167, no. 2, 1966, 309-311

TOPIC TAGS: numerical analysis, Navier Stokes equation, numerical solution, finite difference scheme

ABSTRACT: Two new convergent finite-difference schemes are proposed for solving the three-dimensional boundary-value problem for the system of Navier-Stokes equations

$$\frac{\partial u}{\partial t} - \nu \Delta u + u^k \frac{\partial u}{\partial x_k} = -\text{grad } p + f, \quad (1)$$

$$\text{div } u = 0, \quad u|_S = 0, \quad u|_{t=0} = a,$$

where S is the boundary of the three-dimensional space  $\Omega$ ;  $f = f(x, t)$  and  $a(x)$  are given vectors. A rectangular parallelepipedal lattice

Cord 1/2

UDC: 517.949.8



L 20741-66

ACC NR: AP6010422

with spacing  $h$  and  $\Delta t$  is constructed and a system of equations in  $U_h^i$ ,  $p_h$  ( $i = 1, 2, 3$ ) ( $U_h^i$  and  $p_h$  are difference analogs of function  $U^i$  and  $p$ ) are derived. It is proved that this system of equations has a unique solution on every layer for any given vectors  $f$  and  $a$  and that a sequence of solutions can always be singled out from all solutions of the difference equations derived by the proposed difference schemes which converges to the weak solution (in the sense of E. Hopf) of the boundary-value problem for any relationship between  $h$  and  $\Delta t$ . Orig. art. has: 10 formulas. [LK]

SUB CODE: 12/ SUBM DATE: 05July65/ ORIG REF: 006/ ATD PRESS: 4226

Card

2/2

ACC NR: AT7006687

SOURCE CODE: UR/2517/66/092/000/0093/0099

AUTHORS: Kzhivitskiy, A.; Ladyzhenskaya, O. A.

ORG: none

TITLE: The method of nets for nonstationary Navier-Stokes equations

SOURCE: AN SSSR. Matematicheskii institut. Trudy, v. 92, 1966. Krayevyye zadachi matematicheskoy fiziki (Boundary value problems of mathematical physics), no. 4, 93-99

TOPIC TAGS: Navier Stokes equation, sequence, convergent sequence, vector, Euclidean space, boundary value problem

ABSTRACT: An implicit difference scheme for solving the general nonlinear, non-stationary problem

$$\left. \begin{aligned} \frac{\partial u}{\partial t} - \nu \Delta u + u^k \frac{\partial u}{\partial x_k} &= -\text{grad } p + f, \\ \text{div } u &= 0, \\ u|_S &= 0, \\ u|_{t=0} &= a \end{aligned} \right\} \quad (1)$$

is proposed. Its convergence is investigated. The case of a bounded domain  $\Omega$  and a homogeneous condition is examined. It is shown that the system

Card 1/2

ACC NR: AT7006687

$$u_{\Delta t}^i - \nu u_{\Delta x_k \Delta x_k}^i + \frac{1}{2} u_{\Delta x_k}^{i-1} u_{\Delta x_k}^i + \frac{1}{2} u_{\Delta x_k}^i u_{\Delta x_k}^{i+1} = -p_{\Delta x_k}^i + f_{\Delta x_k}^i$$

$$u_{\Delta x_k}^i = 0, \quad (2)$$

$$u_{\Delta x_k}^i|_{s_k} = 0,$$

$$u_{\Delta x_k}^i|_{t=0} = a_{\Delta x_k}^{i(m)}$$

and

$$\sum_{k=1}^N p_k = 0, \quad k = 1, \dots, N \quad (3)$$

is uniquely solvable in every layer for  $u_h^i$ ,  $p_h$  for any  $f_h$ ,  $a_h$ . It is also shown that from the set of solutions  $\{u_h\}$  constructed according to (2), a sequence can be extracted that converges (when  $\Delta t = ch \rightarrow 0$ ) on a weak solution (in the sense of Hopf) of this problem. In the case of  $n = 2$ , the entire sequence converges on this solution. Orig. art. has: 16 formulas.

SUB CODE: 12/ SUBM DATE: none/ ORIG REF: 006/ OTH REF: 001

Card 2/2

LADYZHENSKAYA, O. I.

Ladyzhenskaya, O. I. and Dudina, D. G. "On the signifincance of the Maksimov reaction under conditions of the operation of the 'venotryads'", Voprosy dermato-venerologii, Vol. IV, 1943, p. 310-11.

SO: U-3736, 21 May 53, (Letopis 'Zhurnal 'nykh Statey, No. 18, 1949).

LADYZHENSKAYA, O. I.

Drozdov, N. P. and Ladyzhenskaya, O. I., and Luzina, V. N. "The cytological picture of urethral pus and morphological changes in Neisser's gonococcus in penicillin therapy," Voprosy dermato-venerologii, Vol. IV, 1948, p. 317-20.

SO: U-3736, 21 May 53, (Letopis 'Zhurnal 'nykh Statey, No. 18, 1949).

LADYZHENSKIY, A.

LADYZHENSKIY, A.

Movable platform for laying cables. Mast, ugl. 6 no.7:12 JI '57.

(MIRA 10:9)

1. Pomoshchnik glavnogo mekhanika shakhty "Rudnichnaya" kombinata  
Molotovugol'.

(Coal mines and mining--Equipment and supplies)

LADYZHENSKIY, A.

Attention designers. Mast. ugl. 7 no. 6:30 Je '58. (MIRA 11:7)

1. Shakhta "Rudnichnaya" tresta Kizelugol'.  
(Mine hoisting)

LADYZHENSKIY, A.

Cutting out rubber packings. Mast. ugl. 7 no. 7:20 J1 '58.

(MIRA 11:8)

1. Pomoshchnik glavnogo mekhanika shakhty "Rudnichnaya" tresta  
Kizelugol'.

(Coal mines and mining--Equipment and supplies)  
(Packings(Mechanical engineering))



LADYZHENSKIY, A., pomeshchnik glavnogo mekhanika

Shock absorber for conveying machinery. Mast. ugl. 7 no. 11:21 N '58.  
(MIRA 11:12)

1. Shakhta "Rudnichnaya" tresta Kizelugel'.  
(Conveying machinery)

LADYZHENSKIY, A.

There is a need for directives on the preparation of technical,  
industrial, and financial plans. Prom. koop. 12 no. 20:10  
0 '58. (MIRA 11:10)

1. Nachal'nik planovo-ekonomicheskogo otдела oblpromsoveta, Voronezh.  
(Industrial management)

LADYZHENSKIY, A.B.

Aggravation of latent tuberculosis in adolescents. Probl.tub.  
no.6:42-47 '61. (MIRA 14:9)

1. Glavnyy vrach Odesskoy detskoy tuberkuleznoy bol'nitsy.  
(TUBERCULOSIS)

1. LADYZHENSKIY, A. M. - DURDENEVSKIY, V. N.
2. USSR (600)
4. Bacterial Warfare
7. The use of bacteriological weapons is a crime against international law.  
Vest.Mosk.un. 7 no. 11, 1952
9. Monthly List of Russian Accessions, Library of Congress, March 1953, Unclassified.



9

CA

Preliminary decarburization of small converter steel with liquid cupola iron. B. N. Lelyzhenskii. *Met. S.* 1125 (1948).—Two methods of preliminary decarburization of steel were tested. In the 1st, after the blow, 1.5% of 75% Mn and 0.8% of 45% FeSi were added, and the converter was shaken for better mixing. The slag was then thickened by addn. of sand and the steel transferred into ladles where it was finally decarburized with 0.1% of Al. In the 2nd process, after the blow, 3% of molten cupola iron was poured slowly into the converter at 1280-1320 (the temp. of the converter steel was 1620-1640) and the converter was shaken. The reaction was at first violent and gradually quieted down. After 20 min. 0.8% of

75% FeMn was added, the slag thickened with sand and the steel transferred into ladles where it was finally decarburized with 0.2% of 45% FeSi and in a few cases with Al. The 1st of these decarburization methods reduced the C content in the converter steel by 0.002% and the 2nd by 0.071%. Half of the latter or 0.035% was removed by C in the pig iron. In the final steel decarburized by the 2nd method, the C was by 0.005% lower than in the steel decarburized by the 1st method. M. Hosh

PA 18/49T89

LADYZHENSKIY, B. N.

USSR/Metals

Steel, Bessemer  
Metallurgy, Ferrous

Dec 48

"Preliminary Decarboxylation of Steel Made by the Short  
Bessemer Process With Molten Cupola Pig," B. N.  
Ladyzhenskiy, Cand Tech Sci, VISHNOM and VINITOL,  
6 pp

"Steel" No 12

Presents results of a comparative study of two  
methods for decarboxilizing Bessemer steel: (1) using  
75% ferromanganese (1.3%), 45% ferrosilicon (0.8%),  
and aluminum (0.1%), and (2) using molten cupola  
pig iron (3%), 75% ferromanganese (0.8%), and 45%  
18/49T89

USSR/Metals (Contd)

Dec 48

ferrosilicon (0.2%). Concludes method (2)  
is feasible. It saves 12 kg ferrosilicon and  
10 kg ferromanganese per ton of usable steel.

*Mr. Zhukov for Prof. Gant. Agre,  
Mechanic 2245,  
ad. Union de la Tech. Soc of  
Techniciens*

18/49T89

LADYZHENSKIY, B. N.

USSR/Metals - Steelmaking

Mar 51

"Intensification of the Steelmaking Process in a Side-Blown Converter," B. N. Ladyzhenskiy, Cand Tech Sci, Al'taysel'mash

"Itay Proizvod" No 3, pp 5-8

Exptl heats were conducted to study effect of ore addn on the blowing period and on decrease in consumption of ferrosilicon. Mechanism of oxidation of Mn, Si and C is similar to that of steel-manufg process in open-hearth or elec furnaces, i.e., atm oxygen is transferred into metal

195T50

BSR/Metals - Steelmaking (Contd)

Mar 51

mainly through ferrous oxide. Addn of ore, increasing the content of ferrous oxide in slag, increases the oxidation rate and shortens the time required for blowing the melt.

195T50

PA 195T50



LADYZHENSKIY, E.N.; ORESHKIN, V.D., kandidat tekhnicheskikh nauk;  
~~SOKHARCHOV~~, Iu.S.; DOBROTVORSKIY, M.M., professor, retsenzent;  
BESSONOV, K.A., dotsent, retsenzent; YERMAKOV, N.P., tekhnicheskiiy redaktor.

[Founding] Liteinoe proizvodstvo. Pod red. V.D.Oreshkina.  
Moskva, Gos. nauchno-tekhn. izd-vo mashinostroit. i sudostroit. (MLRA 7:8)  
lit-ry, 1953. 207 p.  
(Founding)

LADYZHENSKIY, B. N.

Steel - Metallurgy

Problems of smelting steel in side-draft converters. Sel'khoz mashina No. 9, 1953.

Monthly List of Russian Accessions, Library of Congress  
June 1953. UNCL.

LADYZHENSKIY, B.N., kandidat tekhnicheskikh nauk; TUNKOV, V.P., laureat  
Stalinskoy premii, inzhener; BIDULYA, P.N., doktor tekhnicheskikh  
nauk, professor, retsenzent; KONOPASEVICH, V.A., inzhener, redaktor;  
MODEL', B.I., tekhnicheskii redaktor

[Smelting steel for mold casting] Vyplavka stali dlia fasonnogo  
lit'ia. Moskva, Gos. nauchno-tekhn. izd-vo mashinostroit. lit-ry,  
1954. 382 p. (MIRA 7:10)  
(Steel castings)  
(Smelting)

IVANOV, V.G., kandidat tekhnicheskikh nauk; KRYANIN, I.R., kandidat tekhnicheskikh nauk; LADYSHENSKIY, B.N., kandidat tekhnicheskikh nauk.

Overheating of low Bessemer steel. Lit.proizv. no.4:31-32 Ap '56.  
(Bessemer process) (MLRA 9:7)

LADYZHENSKIY, B.N., inzh.

Using low-frequency crucible furnaces. Mashinostroitel' no.1:  
45-46 N '56. (MIRA 12:1)

(Smelting furnaces)

LADYZHENSKIY, BORIS NIKOLAYEVICH

PHASE I BOOK EXPLOITATION

475

Ladyzhenskiy, Boris Nikolayevich and Tunkov, Vladimir Pavlovich

Tekhnologiya izgotovleniya stal'nykh otlivok (Technology of Making Steel Castings) Moscow, Mashgiz, 1957. 255 p. 7,000 copies printed.

Reviewers: Zverev, K.M., Engineer, and Kreshchanovskiy, N.S., Candidate of Technical Sciences; Ed.: Talanov, P.I., Prof.; Ed. of Publishing House: Sirotin, A.I., Engineer; Tech. Ed.: El'kind, V.D.

PURPOSE: This book was written for engineers and technicians in foundry shops and for engineers and designers in the machine-building industry. It may be used as a manual by students studying casting methods.

COVERAGE: The author attempts in this book to discuss the main problems of the casting of various parts for the machine-building industry. These problems, including some theoretical considerations, are reviewed in sequence starting with part design, mold

Card 1/5

# Technology of Making Steel Castings

475

and pattern making, casting, thermal treatment and the repair of flaws in the cast parts. These methods are said to be the most advanced ones and are believed to represent the recent achievements of Soviet scientists and engineers, and the present trend in the Soviet industry. Personalities mentioned are L.N. Podvoysky, who wrote chapter VI, and K.P. Baryshnikov who assisted the author in writing chapter VII. There are 71 Soviet references.

## TABLE OF CONTENTS:

Introduction	3
Ch. I. Fundamentals of Steel Casting Design	5
1. General information	5
2. Technological design considerations	9
3. Structural design features	11
Ch. II. Design of Castings	26
1. General information	26
2. Location of part in the mold	26

Card 2/5

Technology of Making Steel Castings

475

3. Determination of parting line	27
4. Selection of mold preparation methods	28
5. Selection of molding method	28
6. Blue print for casting	29
Ch. III. Fundamentals of Mold Design Technology	37
1. The mold and molding material	37
2. Cores	50
3. Gating	59
4. Risers	72
5. Cooling and insulating materials	103
Ch. IV. Baking of Molds and Cores	114
1. Basic facts about baking of molds and cores	114
2. Methods of baking molds and cores	117
3. Baking conditions for molds and cores	120
Ch. V. Filling of Mold and Cooling of the Casting	126
1. Temperature of poured metal	126
2. Holding the casting in the mold	128
Card 3/5	



Technology of Making Steel Castings

475

Ch. VI. Thermal Treatment of Castings	136
1. Various methods of thermal treatment	136
2. Internal stresses and methods of thermal treatment	140
3. Thermal treatment of carbon steel castings	142
4. Thermal treatment of alloyed steel castings	146
5. Quality control of thermal treatment of castings	152
Ch. VII. Steel Casting Practice	156
1. Bottom-poured stacked mold casting	
2. Manufacture of thin walled castings	161
3. Casting of parts for agricultural machinery	168
4. Casting of fittings	171
5. Casting of parts for tractors	176
6. Casting of parts for transportation machinery	179
7. Castings for heavy machinery	187
8. Casting of forging dies	202
9. Casting of reinforced catings	206
10. Casting of crank shafts	209
Ch. VIII. Special Features of Alloyed Steel Casting	211
1. General information	211
2. Technological and structural considerations in design of castings and in mold making	213
Card 4/5	

Technology of Making Steel Castings	475
3. Defects common to steel alloy castings	216
4. Surface alloying of castings	217
Ch. IV. Defects in Castings, Detection and Repair	219
1. Defects in castings	219
2. Detection of defects	234
3. Eliminating defects	240
Bibliography	252

AVAILABLE: Library of Congress

Card 5/5

GO/ad  
8-26-58

LADYZHENSKIY, B. N.

Distr: 4E2c

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1  
Preliminary decarburization of converter steel. B. N. Ladyzhenskii. U.S.S.R. 106,156, Sept. 25, 1957. After blowing the converter, molten Fe is added and a vacuum is created in the working area of the converter. M. Hosen 11/28

LADYZHENSKIY, Boris Nikolayevich; TUNKOV, Vladimir Pavlovich; ZVEREV, K.M.,  
inzh., retsenzent; KRISHCHANOVSKIY, N.S., kand.tekhn.nauk, retsenzent;  
TALANOV, P.I., prof., red.; SIROTIN, A.I., inzh., red.izd-va;  
BL'KIND, V.D., tekhn.red.

[Technology of preparing steel castings] Tekhnologiya izgotovleniya  
stal'nykh otlivok. Moskva, Gos. nauchno-tekhn. izd-vo mashinostroit.  
lit-ry, 1958. 255 p. (MIRA 11:4)  
(Steel castings)

LADY ZHENSKIY, B.N.

115

PHASE I BOOK EXPLOITATION

SOV/5411

Konferentsiya po fiziko-khimicheskim osnovam proizvodstva stali. 5th,  
Moscow, 1959.

Fiziko-khimicheskiye osnovy proizvodstva stali; trudy konferentsii  
(Physicochemical Bases of Steel Making; Transactions of the  
Fifth Conference on the Physicochemical Bases of Steelmaking)  
Moscow, Metallurgizdat, 1961. 512 p. Errata slip inserted.  
3,700 copies printed.

Sponsoring Agency: Akademiya nauk SSSR. Institut metallurgii imeni  
A. A. Baykova.

Responsible Ed.: A. M. Samarin, Corresponding Member, Academy  
of Sciences USSR; Ed. of Publishing House: Ya. D. Rozentsveyg.  
Tech. Ed.: V. V. Mikhaylova.

Card 1/16

Physicochemical Bases of (Cont.)

SOV/5411

**PURPOSE:** This collection of articles is intended for engineers and technicians of metallurgical and machine-building plants, senior students of schools of higher education, staff members of design bureaus and planning institutes, and scientific research workers.

**COVERAGE:** The collection contains reports presented at the fifth annual convention devoted to the review of the physicochemical bases of the steelmaking process. These reports deal with problems of the mechanism and kinetics of reactions taking place in the molten metal in steelmaking furnaces. The following are also discussed: problems involved in the production of alloyed steel, the structure of the ingot, the mechanism of solidification, and the converter steelmaking process. The articles contain conclusions drawn from the results of experimental studies, and are accompanied by references of which most are Soviet.

Card 2/18

Physicochemical Bases of (Cont.)

SOV/5411

- Ladyzhenskiy, B. N., and M. V. Karakula. Making Low-Carbon Alloyed Steels in Acid Open-Hearth Furnaces 27
- Stroganov, A. I., and A. N. Morozov. Behavior of Chromium in the Bath of a Basic Open-Hearth Furnace 39
- Petukhov, B. G. Making Chromium-Nickel Steels in Large Open-Hearth Furnaces With the Use of Nickel Oxide 46
- Omarov, A. K., and A. Ye. Khlebnikov. Intensifying the Working Period of the Open-Hearth Scrap Process 54
- [ The following persons participated in the research work:  
Engineer Munasypova, Engineer T. Kovaleva, and Technicians  
U. Rakhmanulov, V.V. Ponomareva, L. Rusnyak, Z. Zaporozhan,  
A. Perkova, S. Bilyalova, and V. Guseva.]

Card 4/16

GOROZHANKIN, A.N., kand.tekhn.nauk; NOVITSKIY, V.K., kand.tekhn.nauk;  
 KRYANIN, I.R., doktor tekhn.nauk; IODKOVSKIY, S.A., kand.tekhn.  
 nauk; LADYZHENSKIY, B.N., kand.tekhn.nauk; MIL'MAN, B.S., kand.tekhn.  
 nauk; KLOCHNEV, N.I., kand.tekhn.nauk; TSYPIN, I.O., kand.tekhn.  
 nauk; LEVIN, M.M., kand.tekhn.nauk; BALDOV, A.L., inzh.; LYASS,  
 A.M., kand.tekhn.nauk; CHERNYAK, B.Z., kand.tekhn.nauk; ASTAF'YEV,  
 A.A., kand.tekhn.nauk; YERMAKOV, K.A., inzh.; GRIBOYEDOV, Yu.N.,  
 kand.tekhn.nauk; MYASOYEDOV, A.N., inzh.; BOGATYREV, Yu.M., kand.  
 tekhn.nauk; UNKSOV, Ye.P., doktor.tekhn.nauk, prof.; SHOFMAN, L.A.,  
 kand.tekhn.nauk; PERLIN, P.I., inzh.; MOSHNIN, Ye.N., kand.tekhn.  
 nauk; PROZOROV, L.V., doktor tekhn.nauk; CHERNOVA, Z.I., tekhn.  
 red.

[Some technological problems in the manufacture of heavy machinery]  
 Nekotorye voprosy tekhnologii tiazhelego mashinostroeniya. Moskva,  
 Gos.nauchno-tekhn.izd-vo mashinostroit. lit-ry. Part 1. [Steel smelt-  
 ing and casting, founding, heat treatment, shaping metals by pres-  
 sure] Vyplavka i razlivka stali, litelnoe-proizvolstvo, termiche-  
 skaya obrabotka, obrabotka metallov davleniem. 1960. 266 p. (Moscow.  
 Tsentral'nyi nauchno-issledovatel'skii institut tekhnologii i mashi-  
 nostroeniya. [Trudy] no. 98). (MIRA 13:7)  
 (Steel) (Founding) (Forging)



LADYZHENSKIY, Boris Nikolayevich; BASHMAKOV, Aleksandr Dmitriyevich;  
POZDNYAKOVA, G.L., red. izd-va; VENETSKIY, S.I., red. izd-va;  
OBUKHOVSKAYA, G.P., tekhn. red.

[Treatment of liquid metals by powder in a gas stream] Obrabotka  
zhidkogo metalla poroshkami v strue gaza. Moskva, Gos. nauchno-  
tekhn. izd-vo lit-ry po chernoi i tsvetnoi metallurgii, 1961. 115 p.  
(MIRA 14:12)

(Powder metallurgy) (Liquid metals)

18.3200

22572  
S/133/61/000/001/003/016  
A054/A033

AUTHORS: Ladyzhenskiy, B.N., Candidate of Technical Sciences; Bashmakov, A.D.  
Engineer

TITLE: The Dependence of Metal-Desulfurization on the Conditions of Mass  
Transfer

PERIODICAL: Stal', 1961, No. 1, pp. 29 - 30

TEXT: At steel melting temperatures chemical reactions take place at high velocities. The only factor limiting the reaction speed is the mass transfer at the place of reaction depending - among other things - on the temperature conditions, the diffusion of the reacting substances, the size of surface on which the reactions take place and on the layer thickness. Evidently, by improving these conditions, several metallurgical processes could be accelerated. Based on the above considerations and tests, satisfactory results have been obtained by using powdery materials during the melting in hearth-type furnaces, for the purpose of accelerating the desulfurization of the metal which, under normal conditions, is extremely slow (0.00007 - 0.00125% S/min). This is mainly due to the small reaction area between the metal and the slag relative to the weight unit of the metal

Card 1/6

22572

S/133/61/000/001/003/016

A054/A033

# The Dependence of Metal-Desulfurization on the Conditions of Mass Transfer

(S/T), for which the following values have been established:

Furnace	S : T, m <sup>2</sup> /t
15-ton open-hearth furnace.....	0.9
125-ton open-hearth furnace .....	0.4
arc furnace .....	~0.8
induction furnace .....	~4.2

An increase in this specific contact surface not only enlarges the reaction area but also increases the thickness of the layers taking part in the reaction which also contributes to accelerating the mass transfer at the place of reaction. Blowing powdery materials, finely crushed slag-forming substances by a gas jet into the liquid metal in the ladle, the desulfurization speed of the metal increased to 0.005% S/min (Ref. 1, B. Ladyzhenskiy and N. Sashchikhin, ITEIN, No. 743, 1960). By blowing powdery fluxing agents with a specific surface of 435 cm<sup>2</sup>/100 g into the metal in amounts of 5% of the weight of the metal to be blown through, the reaction area can be enlarged to 200 m<sup>2</sup>/ton and the desulfurization rate can be raised to 0.2% S/min. The effect of the reaction surface of the phases on the desulfurization rate is verified by an analysis of the equilibrium condition of sulfur in the metal-slag system (Ref. 2, Fischer and Spitzer, Archiv f. d. Eisenhüt-

Card 2/6

22572

S/133/61/000/001/003/016

A054/A033

# The Dependence of Metal-Desulfurization on the Conditions of Mass Transfer

tenwesen, 1958, No. 9) (Fig. 1), for the conventional desulfurization process and also for the new method, using pulverous substances. In the first case the metal was melted in a 12-kg lime-dolomite crucible of an induction furnace, containing 0.030% S. After adding lime it was held under slag at 1,600°C for two hours. In the second case the metal was melted in a 50-kg magnesite crucible of the induction furnace, heated up to 1,700°C, blown through with a mixture of 55% CaO, 40% CaC<sub>2</sub> and 5% Al. The quantity of mixture employed amounted to 4.5 % of the metal weight with a temperature drop of 200°C during the blowing process. Nitrogen was used as carrier gas. Figure 1 shows that the S-equilibrium in the metal-slag system is attained in 50 - 60 min in the conventional process, whereas in the new process it takes only 2.5 min to reach this point. Another feature of mass transfer influence on desulfurization is the fact that slag and slag-forming substances are more fully utilized in separating sulfur from the metal. In Figure 2 comparison is made on the relationship between the distribution coefficient of sulfur in the metal-slag system and the basicity of the slag. By enlarging the specific contact area between metal and slag, the amount of sulfur separated from the metal increases, the basicity of the slag remaining the same. The minimum degree of sulfur removal in the open-hearth process corresponds to an S/T value between 0.4.

Card 3/6

22572

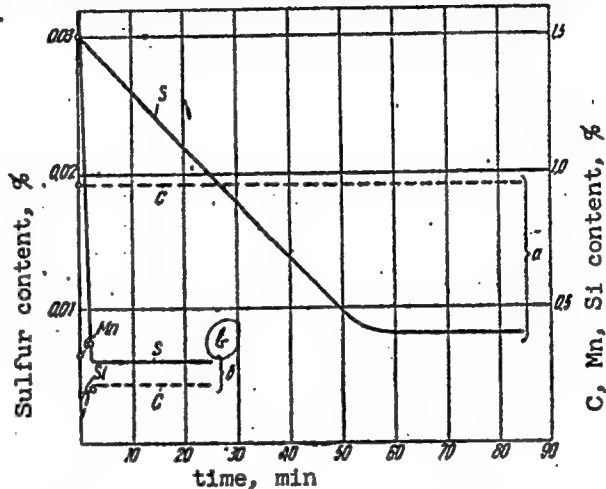
S/133/61/000/001/003/016  
A054/A033

# The Dependence of Metal-Desulfurization on the Conditions of Mass Transfer

- 0.9 m<sup>2</sup>/ton, while the maximum is attained in the process of blowing through the metal finely crushed powdery mixtures, for which S/T exceeds 200 m<sup>2</sup>/ton. There are 2 figures and 2 references; 1 Soviet and 1 Non-Soviet.

ASSOCIATION: TsNIITMASH

Figure 1: Establishing the sulfur equilibrium in the metal-slag system with various methods of desulfurization. a - holding the metal under lime slag (Ref. 2); b - blowing powdery mixtures, in a nitrogen gas current, into the metal.



Card 4/6

VLASOV, V.I.; KOMOLOVA, Ye.F.; LADYZHENSKIY, B.N., kand. tekhn.  
nauk, retsenzent; MARKIŽ, Yu.L., inzh., red.izd-va;  
SMIRNOVA, G.V., tekhn. red.

[Cast G13L high-manganese steel; properties and manufac-  
ture] Litaia vysokomargantsovistai stal' G13L; svoistva  
i proizvodstvo. Moskva, Mashgiz, 1963. 194 p.

(MIRA 16:6)

(Manganese steel) (Steel castings)

ANIDALOV, M.P.; LADYZHENSKIY, B.N.

Organizing the operation of electric melting furnaces in mass  
production foundries. Lit. proizv. no.10:11-12 0 '63. (MIRA 16:12)

LADYZHENSKIY, B.N.; KULINICH, V.P.; KATEYEV, Yu.V.; ZARUBIN, S.N.; ROZENBLIT, Ya.L.; ABROSIMOV, V.I.

Desulfuration of acid electric steel by the blowing-in of powderlike limestone. Lit. proizv. no.8:42-43 Ag '64. (MIRA 18:10)



GLUSHCHENKO, V.G.; LADYZHENSKIY, B.N., kand. tekhn. nauk; YEFREY, G.I.,  
kand. tekhn. nauk

Using scrap metal in the side-blown oxygen converter process.  
Nat. 1 gornorud. prom. no.6:21-23 N-D '65. (MIRA 18:12)

YAKOVLEV, Nikolay Nikolayevich; GLUSHCHENKO, Viktor Grigor'yevich;  
LADYZHENSKIY, B.N., retsenzent

[Steel production in small converters] Proizvodstvo stali  
v malykh konverterakh. Moskva, Metallurgiya, 1965. 142 p.  
(MIRA 18:7)

LADYZHENSKIY, B. V. and TUNKOV, V. P.

"Steel Smelting for Shaped Casting," Sci. and Tech. State Publ. House for Literature on Machine Construction, Moscow, 1954

Translation of Table of Contents and summary of context - D 257848, 6 Jul 55

LADYZHENSKIY, G.N. [Ladyzhens'kyi, H.M.]; KIRICHENKO, I.P. [Fyrychenko, I.P.]

Mineral composition, minor elements, and the structure of the  
Upper Cretaceous and Paleogene shells and skeletons of marine  
organisms in Bakhchisaray District of Crimea Province. Dcp. AN  
URSR no.7:907-910 '65. (MIRA 18:8)

1. L'vovskiy gosudarstvennyy universitet.

KRASNOSHEL'SKIY, M.A.; LADYZHENSKIY, L.A.

Conditions for total continuity of P.S.Urysohn's operator valid  
in the space  $L^p$ . Trudy Mosk.mat.ob-va 3:307-320 '54. (MLRA 7:7)  
(Operators (Mathematics)) (Spaces, Generalized)

KRASNOSEL'SKIY, M.A.; LADYZHENSKIY, L.A.

Structure of the spectrum of positive heterogeneous operators.  
Trudy Mosk.mat.ob-va 3:321-346 '54. (MIRA 7:7)  
(Operators (Mathematics) (Topology))

LADYZHENSKIY, L. A.

USSR/Mathematics

Card : 1/1

Authors : Ladyzhenskiy, L. A.

Title : General conditions of complete continuity of the P. S. Uryson operator effective in space of continuous functions.

Periodical : Dokl. AN SSSR, 96, Ed. 6, 1105 - 1108, June 1954

Abstract : The general conditions of complete continuity of a Uryson operator are described. The most natural and simple satisfactory condition for complete continuity of the U-operator in space C consists in that that the function  $K(x, y, u)$  is continuous in all variables combined. Another source showed satisfactory conditions for complete continuity of the U-operator in space C without assuming the reticence of the set G. Five references.

Institution : The Mining Institute, Molotov

Presented by : Academician P. S. Aleksandrov, April 10, 1954

✓ Lashitskii, L. A. On a class of non-linear equations.

Voronets, Gos. Univ. Trudy Sem. Funktsional. Anal.  
no. 2 (1956), 31-38. (Russian)

Motivated by the need to study operators  $A$  of the form:  
 $A\varphi(x) = \int K(x, y, \varphi(y))dy$ , the author continues his in-  
vestigations in the field of operators over partially ordered  
Banach spaces. Included are two types of theorems, those  
on the non-existence and those on the existence of so-  
lutions of equations of the form  $A\varphi + I = \lambda\varphi$ ,  $I\varphi = f$  in the  
positive cone  $K$  which defines the partial ordering in the  
space  $E$ . For example: (1) Let  $A\varphi \leq Q\varphi$ ,  $\varphi \in K$ , where  $Q$  is  
a linear, completely continuous  $u_0$ -bounded ( $\exists u_0 \in K$   
( $u_0 \neq 0$ ) such that  $0 \leq \varphi \leq Ku \Rightarrow 3$  an integer  $p$  and  $\alpha, \beta > 0$   
such that  $\alpha u_0 \leq Q^p \varphi \leq \beta u_0$ ) operator. If  $\lambda_0$  is a positive  
eigenvalue of  $Q$ , then  $A\varphi + I = \lambda\varphi$ ,  $I \neq 0$ , has no solution in  
 $K$  if  $\lambda < \lambda_0$ . (2) In the same notation, if  $A$  is asymptotic to  
 $Q$  ( $\lim_{\|\varphi\| \rightarrow \infty} \|A\varphi - Q\varphi\|/\|\varphi\| = 0$ ) then there are positive  
solutions of the equation if  $\lambda > \lambda_0$ .

There are also sections on  $u_0$ -concave operators and on  
specializations in which the partial ordering is continuous.

B. Gelbaum (Minneapolis, Minn.).

2  
1-FW

1/8 MW



SUBJECT USSR/MATHEMATICS/Functional analysis CARD 1/2 PG - 744  
 AUTHOR LADYZENSKIY L.A.  
 TITLE On non-linear equations with positive non-linearities.  
 PERIODICAL Uspechi mat.Nauk 12, 1, 211-212 (1957)  
 reviewed 5/1957

The author investigates the positive solutions of the equations

$$(1) \quad \varphi = \lambda \Delta \varphi$$

and

$$(2) \quad \varphi = \lambda \Delta \varphi + f$$

in a Banach space with a cone, where  $\Delta$  is a non-linear operator and  $\Delta \theta = \theta$ . It is assumed that  $\Delta$  has the following property: In the cone  $K$  there exists an element  $u_0$  such that for every  $\varphi \gg \theta$  ( $\varphi \neq \theta$ )

$$\mu u_0 \gg \Delta \varphi \gg \nu u_0$$

( $\mu = \mu(\varphi)$  and  $\nu = \nu(\varphi)$  are positive numbers) and for every  $\varphi \gg \gamma u_0$  ( $\gamma > 0$ ) and arbitrary numbers  $a$  and  $b$  ( $0 < a < b < 1$ ) there holds the relation

$$\Delta(t\varphi) \geq (1+\eta)t\Delta\varphi, \quad (a \leq t \leq b), \quad \eta = \eta(a, b, \varphi) > 0.$$

Under this and some further less essential conditions it is shown 1) that for

Uspechi mat.Nauk 12, 1, 211-212 (1957)

CARD 2/2

PG - 744

$\lambda \in (\lambda_0, \lambda_\infty)$  there exist positive solutions of (1) and for  $\lambda \in (\lambda_0, +\infty)$  there exist positive solutions of (2) and that they are unique; 2) that for other  $\lambda$ -values (1) and (2), respectively, have no positive solution being different from zero; 3) that the positive solutions of (1) and (2) depend continuously on  $\lambda$  and they increase monotonely with  $\lambda$ . Besides a method for the determination of  $\lambda_0$  and  $\lambda_\infty$  is given.

A detailed representation of these and similar results is contained in the author's thesis (Kasanj, 1954).

11

16(1)

05256

AUTHORS: Krasnosel'skiy, M.A., and Ladyzhenskiy, L.A. SOV/140-59-5-12/25

TITLE: On the Extent of the Notion  $u_0$ -Concave Operator

PERIODICAL: Izvestiya vysshikh uchebnykh zavedeniy. Matematika, 1959, Nr 5, pp 112-121 (USSR)

ABSTRACT: The authors consider

$$(1) \quad A\varphi(x) = \int_F G[x, y, \varphi(y)] dy.$$

An operator  $A$  in the Banach space  $E$  which is partially ordered with the aid of a cone  $K$ , is called  $u_0$ -concave if it is positive and monotone and if there exists a positive element  $u_0$  so that:

1) For every  $\varphi \in K$  ( $\|\varphi\| \neq 0$ ) there exist  $\alpha, \beta$ , so that

$$(2) \quad \alpha u_0 \leq A\varphi \leq \beta u_0.$$

2) For every  $\varphi \in K$  for which  $\varphi \geq \gamma u_0$  ( $\gamma > 0$ ), and arbitrary  $0 < a < b < 1$  there exists an  $\eta$  so that:

$$(3) \quad A(t\varphi) \geq (1+\eta)tA\varphi \quad (a \leq t \leq b)$$

(the sign  $\leq$  is also used for marking the ordering relations).

In the present paper the authors give conditions for the  $u_0$ -

Card 1/2 concavity, e.g.: For an increasing  $u$  let  $G(x, y, u)$  be increasing,

05256

On the Extent of the Notion  $u_0$ -Concave Operator

SOV/140-59-5-12/25

$G(x,y,0) \equiv 0$ . Let  $H(x,y,u) = \frac{1}{u} G(x,y,u)$ ;  $u_0 = u_0(x) \equiv 1$ .

Theorem 1: Let the operator (1) act in the space  $C$  of functions continuous on a bounded, closed set  $F$  of the Euclidean space. Let  $H(x,y,u)$  be not increasing with respect to  $u$  and

(4)  $H(x,y,u_1) - H(x,y,u_2) > 0$

for almost all  $y \in F$ . Then (1) is  $u_0$ -concave in  $C$  with respect to the cone of all non-negative functions.

The authors formulate 8 theorems. They mention P.S.Uryson, I.A. Bakhtin, and Ya.D.Mamedov.

There are 8 Soviet references.

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Card 2/2